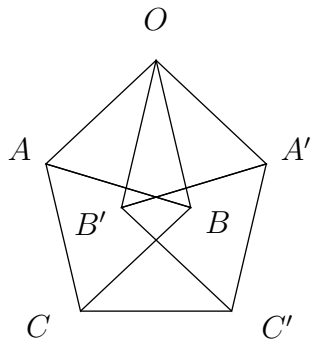
Mathematical olympiad of Baltic Sea schools 2011
2nd year of upper secondary school

1. Aino celebrated his one-year-birthday on Friday, the 11th of November this year. Which day of the week will be her 50th birthday?

2. The three consecutive terms r_i, r_{i+1} and r_{i+2} of the geometric progression $(r_1, r_2, \ldots, r_i, \ldots)$ satisfy the equality $r_{i+2} = r_{i+1} + 6r_i$. In addition, $r_1 = 1$. Determine the general term r_i.

3. Prove that for an arbitrary real number x we have that
$$\sin\left(\frac{1}{2}\sin x\right) < \cos\left(\frac{1}{2}\cos x\right).$$

4. The configuration below is called *the Moser spindle*. All the designated line segments drawn in the figure have length one. Determine the distance between points B and B'.



5. Three mathematicians wearing hats are sitting behind each other in a bus. They have chosen their hats from a wardrobe knowing that 3 black and 2 red hats are kept there. Each of the mathematicians sees only the hats of other mathematicians sitting in front of her/him, in particular, no one sees the color of her/his own hat. When the mathematician in the back row is asked whether he knows the colour of his hat, (s)he answers in the negative. The mathematician sitting in the middle overhears the conversation and says that he does not know the colour of her/his hat, either. Explain why the mathematician in the front now knows the colour of her/his hat.