

Mathematical olympiad of Baltic Sea schools 2011
1st year of upper secondary school

1. The roots of the equation $x^2 + 4x - 7 = 0$ are x_1 and x_2 . The roots of the equation $x^2 + ax + b = 0$ are the inverses x_1^{-1} and x_2^{-1} , respectively. Determine b .

2. The expression

$$\frac{(x^3 + 8)^2}{x + 2}$$

reduces to a polynomial. What is the sum of the coefficients of that polynomial?

3. Is it possible that the number $x^2 + y^2 - 2011$ is divisible by four, for integers x and y ?
4. Show that

$$\frac{1^1 \cdot 2^2 \cdot 3^3 \cdot 4^4 \cdot 5^5 \cdot 6^6 \cdot 7^7 \cdot 8^8 \cdot 9^9 \cdot 10^{10} \cdot 11^{11} \cdot 12^{12} \cdot 13^{13}}{1^{13} \cdot 2^{12} \cdot 3^{11} \cdot 4^{10} \cdot 5^9 \cdot 6^8 \cdot 7^7 \cdot 8^6 \cdot 9^5 \cdot 10^4 \cdot 11^3 \cdot 12^2 \cdot 13^1}$$

is an integer.

5. The configuration below is called *the Moser spindle*. All the designated line segments drawn in the figure have length one. Determine the area of the triangle OCC' .

